positivity

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Abstracts

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List of plenary talks

List of contributed talks

Plenary Talks

Completeness for vector lattices

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We will deal in this talk with several kinds of completeness that a vector lattice can have. Connection between them is studied. The main result is the equivalence between uo-completeness and universal completeness. Several applications of this result will be given. A particular attention will be paid to the notion of sup-completion.

How to be positive in natural sciences?

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In many fields of science there is the chicken or the egg dispute – whether applications drive theory, or the theory makes applications possible. Actually, in mathematics, there is another option, when both the applied and pure parts of a certain theory function happily oblivious of each other.

In my opinion, the idea of positivity in Banach spaces, along with compactness, plays the role of an important bridge between the finite and infinite dimension, allowing for a number of concepts from the undergraduate calculus, like (part of) the Bolzano-Weierstrass theorem, or the Lyapunov stability theorem, to find their place in the Banach space theory.

The talk will present a number of applications of such concepts to some models arising in natural sciences and analysed within the framework of the theory of semigroups of operators.

Real positivity and maps on operator algebras

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We begin by reviewing the theory of real positivity initiated by the speaker and Charles Read, and present many new general results about real positive maps. The key point is that real positivity is often the right replacement in a general algebra A for positivity in C^*-algebras. We then apply this to contractive projections and isometries of operator algebras. For example we describe recent joint work with Matt Neal in which we generalize and find variants of certain classical results on contractive positive projections on C^*-algebras and JB algebras due to Choi, Effros, Stormer, Friedman and Russo, and others. In previous work with Neal we had done the completely contractive' case. We also give a new Banach-Stone type theorem for isometries between our algebras, and an application of this is given to the characterization of 'symmetric projections'. In the last part of the talk, joint with Louis Labuschagne, we focus on a special case of the projections considered above that we consider to be a good noncommutative generalization of the 'characters' (i.e. homomorphisms into the scalars) on an algebra. We consider and solve several problems that arise when generalizing classical function algebra results involving characters. For example, the Jensen inequality, a related theorem of Bishop et al concerning Jensen measures, and the theory of Gleason parts. Our approach to the latter topics involves e.g. Brown measure on von Neumann algebras (for our noncommutative Jensen measures), and the noncommutative (operator theoretic) hyperbolic distance/Schwarz-Pick inequality (for our Gleason/Harnack parts). Some of the results in the talk constitute work in progress, and as we proceed we list many interesting open questions.

Lattice homomorphisms in harmonic analysis

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If G is a locally compact group, then natural spaces such as $\mathrm{L}^1(G)$ or $\mathrm{M}(G)$ carry more structure than just that of a Banach algebra. They are also vector lattices, so that they are, in fact, Banach lattice algebras. Therefore, if they act by convolution on, say, $\mathrm{L}^p(G)$, it is a meaningful question to ask if the corresponding map into the Banach lattice algebra $\mathrm{L}_\mathrm{r}(\mathrm{L}^p(G))$ of regular operators on $\mathrm{L}^p(G)$ is not only an algebra homomorphism, but also a lattice homomorphism. Analogous questions can be asked in similar situations, such as the left regular representation of $\mathrm{M}(G)$.

In this lecture, we shall give an overview of how these questions have been tackled, and how this finally resulted in the rule of thumb that the answers tend to be affirmative whenever the questions are meaningful.

This is joint work with Garth Dales and David Kok.

Perron–Frobenius Theory and the Asymptotics of Linear Dynamics

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During more than a century since the ground-breaking work of Perron and Frobenius, spectral theory of positive matrices and operators has evolved into a large and diverse subfield of mathematical analysis. By providing tremendous insights into the spectral structure of positive operators, this theory helps us to understand the long-time behaviour of linear dynamical systems occurring in various disciplines such as Markov processes, partial differential equations and mathematical biology.

We give a survey of this interaction between Perron–Frobenius theory and the long-term behaviour of dynamical systems. Starting with an outline of Perron's and Frobenius' original results on matrices we quickly proceed to infinite dimensional realms, where various applications to positive operator semigroups await us along with a notorious open problem. After comparing a few techniques and methods, we conclude by presenting some recent advances in the asymptotic theory of positive semigroups.

101 Years of vector lattice theory, P.J. Daniell: A general form of the integral

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The consentient wisdom about the history of the theory of vector lattices is that it was founded by F. Riesz, H. Freudenthal and L.V. Kantorovitch in the years around 1935. It is believed that the theory was initiated by a short note delivered to the

International Mathematical Congress at Bologna in 1928 by F. Riesz, the results of which were published in a 1940 paper in the Annals of Mathematics (see the well known textbooks by W.A.J. Luxemburg and A.C. Zaanen [3], H.H. Schaefer [5], C.D. Aliprantis and O. Burkinshaw [1]).

However, 10 years earlier in 1918, P.J. Daniell published his remarkable and influential paper "A general form of integral" in the Annals of Mathematics [2]. In this paper he not only defined a Riesz space with order unit, but he showed that an order bounded σ-order continuous linear functional on the space has an absolute value and can be decomposed into its positive and negative parts, each of which is a positive σ-order continuous functional (these positive functionals are called positive integrals, or I-integrals, and the order bounded functional is called an S-integral).

A stochastic integral is a vector valued linear operator. This opened the possibility, pursued by P. Protter [4], to follow Daniell's method to define stochastic integrals ignoring the possible existence of measures.

In this talk we follow Protter and apply Daniell's method to define the stochastic integral in abstract Riesz spaces. Contrary to Potter, we do not use a metric topology, but only Riesz space conditions. We conclude by applying the result to define a stopped process with reference to a stopping time.

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The Role of Positivity in Mathematical Economics: Monotonicity and Free-Disposal in Walrasian Equilibrium Theory

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This is joint work with Niccolò Urbinati (Dipartimento di Scienze Economiche e Statistiche, University of Napoli Federico II, Complesso Monte S. Angelo, Via Cintia, Napoli 80126, Italy)

Halmos (1956) writes:

> It has often happened that a theory designed originally as a tool for the study of a physical problem came subsequently to have purely mathematical interest. When that happens the theory is generalized way beyond

> the point needed for applications, the generalizations make contact with other theories (frequently in completely unexpected directions), and the subject becomes established as a new part of pure mathematics. The part of pure mathematics so created does not (and need not) pretend to solve the physical problem from which it arises; it must stand and fall on its own merits. Physics is not the only external source of mathematical theories; other disciplines (such as economics and biology) can play a similar role.

In this broad-ranging talk, I shall draw on recent applications of ordered structures in decision theory, and as it feeds into Walrasian general equilibrium and game theory. I shall draw on [2] [1]

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Pre-Riesz spaces: embeddings and structure preserving operators

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We discuss the embedding of partially ordered vector spaces into Riesz spaces, where we compare the Dedekind completion, the Riesz completion and (for order unit spaces) the functional representation. Pre-Riesz spaces are the partially ordered vector spaces that embed order densely into Riesz spaces, their so-called vector lattice covers. Concepts from Riesz space theory such as disjointness, ideals, and bands are extended to pre-Riesz spaces, as well as disjointness preserving operators. In particular, Riesz* homomorphisms on pre-Riesz spaces are disjointness preserving. We present related notions and examples. Every result on disjointness preseving operators in Riesz spaces leads to the question whether a similar result is true in pre-Riesz spaces. As an instance, recall that the inverse of a disjointness preserving bijection between Banach lattices is again disjointness preserving. In pre-Riesz spaces, a corresponding result is valid in finite dimensions. To obtain similar results in infinite dimensional pre-Riesz spaces, appropriate extensions of norms to vector lattice covers are needed.

Kalauch, A., van Gaans, O.: Pre-Riesz Spaces. De Gruyter, 2018.

Cones as symmetric spaces

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A cone C, with non-empty interior, in a finite dimensional vector space V is said to be symmetric if it is self-dual under some inner-product on V, and the group of linear automorphisms of C acts transitively on the interior of C. Such cones are very special. Indeed, the famous Koecher-Vinberg Theorem says that the symmetric cones are precisely the cones of squares in Euclidean Jordan algebras. Moreover, symmetric cones can be equipped with a Riemannian metric turning them into Riemannian symmetric spaces.

Recent works indicate that the connection between Jordan algebras and the geometry (or symmetry) of cones exists more generally in, possibly infinite dimensional, complete order unit spaces. In this talk I will give an overview of the existing results in this area and discuss some of the ideas from metric geometry of cones that play a fundamental role.

Fredholm and r-Fredholm theory in ordered Banach algebras

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The introduction of Fredholm theory relative to general unital homomorphisms $T : A \to B$ between Banach algebras A and B was due to Robin Harte [7], after which this investigation was continued by several authors. Motivated by results of Egor Alekhno in [1] and [2], the *upper Browder* elements in an ordered Banach algebra (OBA) were introduced in [3], thereby initiating the study of the interplay between Fredholm theory and ordering. One of the problems that arose was to study conditions under which the spectral radius of a positive element a lying outside the Fredholm spectrum of a will be outside the upper Browder spectrum of a as well. This property is referred to as the *upper Browder spectrum property.* Some solutions to this problem were given in [4].

Inspired by an indication that these elements could be useful in the context of OBAs, certain variants of the invertible, Fredholm and Browder elements (namely the r-invertible, r-Fredholm and r-Browder elements) were investigated in [5] in the context of general Banach algebras. Continuing the development of Fredholm theory in ordered Banach algebras, the notions of *upper r-Browder* and *contractive upper r-Browder* elements in an OBA were introduced in [6] and, motivated by previous results, conditions under which the positive almost r-invertible r-Fredholm elements are contractive upper r-Browder were studied. This, in turn, led to the recovering (and strengthening) of certain results regarding the upper Browder spectrum property.

In this talk, based on joint work with Ronalda Benjamin and in part on joint work with both Ronalda Benjamin and Niels Laustsen, I will give an overview of some of the development of the above theory.

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Bibasic sequences in Banach lattices

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In this joint project with M.Taylor, we consider Schauder basic sequences in Banach lattices for which the expansion of a vector converges not only in norm but also in order. We call such sequences *bibasic*. The concept of a bibasic sequence was originally introduced by Gumenchuk et al in 2015.

We present several equivalent characterizations of bibasic sequences. We discuss various properties and examples of bibasic sequences. We also consider several special types of bibasic sequences, including *permutable* sequences, i.e., sequences for which every permutation is bibasic, and *absolute* sequences, i.e., sequences where expansions remain convergent after we replace every term with its modulus.

Conditional expectation operators in Riesz spaces

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Researchers over the past 46 years have given various definitions for conditional expectation operators (and hence filtrations and stochastic processes) in the Riesz space setting. Each of these definitions has it strengths and weaknesses, however some of the most important points to consider when making a choice of definition are: when the Riesz space is $L^1(\Omega, \mathcal{A}, P)$ does the definition give a classical conditional expectation operator or a generalization there of; does the definition yield a strong enough structure to be interesting and yield a workable theory of stochastic processes; it the definition general enough to yield more than classical $L^1(\Omega, \mathcal{A}, P)$ theory. In this talk these definitions will be reviewed along with their resulting martingale theory and its relationship to classical $L^1(\Omega, \mathcal{A}, P)$ theory. Some of this work yields interesting results when taken back to probability space processes.

The content of this talk has been based on joint work with Wen-Chi Kuo, Coenraad Labuschagne, David Rodda and Michael Rogans but also depends on and relates to the work of many other researchers.

A local Hahn-Banach theorem and its applications

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An important consequence of the Hahn-Banach Theorem says that on any locally convex Hausdorff topological space X, there are sufficiently many continuous linear functionals to separate points of X. In this talk, we will establish a "local" version of this theorem. The result is applied to the study of the uo-dual of a Banach lattice. We will also discuss some open problems on local convexity in $\mathbb{L}^0$ and switching probability measures.

The talk is based on joint worth with Niushan Gao and Denny Leung.

Contributed Talks

On discretization of positive systems

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A system $\dot{x} = f(x)$ is positive if for every initial state $x(0) \in \mathbb{R}^n_+$ the forward trajectory $x(t)$, $t \geq 0$, stays in the cone $\mathbb{R}^n_+$. This definition may be extended to a system on a time scale $\mathbb{T}$ described by $x^\Delta(t) = f(x(t))$, where $x^\Delta(t)$ is the delta derivative of x at time t [1]. If the graininess $\mu(t)$ is positive for all $t \in \mathbb{T}$ (i.e. $\mathbb{T}$ is a discrete time scale), then $x^\Delta(t) = (x(t + \mu(t)) - x(t))/\mu(t)$. By discretization of the system $\dot{x} = f(x)$ we mean a system $x^\Delta(t) = f(x(t))$ on some time scale with a positive (not necessarily constant) graininess function μ. A discretized system coming from a positive system does not have to be positive. However we can show the following:

Theorem If the system $\dot{x} = Ax$ is positive and exponentially stable and its discretization $x^\Delta = Ax$ is positive, then $x^\Delta = Ax$ is also exponentially stable.

A similar fact holds for positive nonlinear systems $\dot{x} = f(x)$, with f of class C^1, under some additional assumptions. These results follow from characterizations of exponential stability obtained in [2, 3].

This work was supported by the National Science Centre under the grant No. 2017/25/B/ST7/01471.

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A spectral mapping theorem for the upper Weyl spectrum

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Since its inception, Fredholm theory has become an important aspect of spectral theory. Among the spectra arising within Fredholm theory is the Weyl spectrum, which has been intensively studied by several authors, both in the operator case and in the general setting of Banach algebras [2].

In order to investigate possible connections between Fredholm theory and positivity in arbitrary ordered Banach algebras (OBAs), research which begun in [1] in the context of bounded linear operators on Banach lattices, we introduce the concept of 'upper Weyl spectrum' for an element of a general OBA. (This spectrum is generally larger than the well-known Weyl spectrum.)

In this talk we will discuss the spectral mapping properties for the upper Weyl spectrum of an arbitrary OBA element [3].

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Some results about the lattice of closed ideals of $\mathcal{L}^{\nabla}(\mathcal{X})$ for certain X's

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An important feature of any Banach algebra is its lattice of closed ideals. Over the past 20 years, significant progress has been made in the study of the ideal structure of $\mathcal{B}(\mathcal{X})$ (the Banach algebra of bounded linear operators on a Banach space X) for various kinds of Banach lattices X. In the case of $\mathcal{L}^{\nabla}(\mathcal{X})$, its order counterpart, it has long been known that the 'right' class of closed algebra ideals to consider are those which are also ideals in the order sense. Little has been done since the 80's concerning the lattice of closed order and algebra ideals of $\mathcal{L}^{\nabla}(\mathcal{X})$. In my talk I shall report on some recent results in this direction.

Positive Miyadera–Voigt perturbations of bi-continuous semigroups

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Consider the Banach space of bounded continuous function $\mathrm{C_b}(\mathbb{R})$ and the translation semigroup $(T(t))_{t\geq 0}$ defined by $(T(t)f)(s) := f(s+t),\ t \geq 0,\ f \in \mathrm{C_b}(\mathbb{R}),\ s \in \mathbb{R}$. This semigroup fails to be strongly continuous with respect to the Banach space norm. Nevertheless $(T(t))_{t\geq 0}$ is strongly continuous with respect to a weaker locally convex topology. This leads to the notion of bi-continuous semigroups which theory was developed by Kühnemund [4]. This theory has the advantage that not only Markov transition semigroup, but also semigroups induced by jointly continuous flows or implemented semigroups, just to mention a few, can be handled in a unified manner. Perturbations in this context where studied by Farkas [3] and more recentely by Budde and Farkas [2]. By taking positivity into account there are several results involving positive perturbations for C_0-semigroups for example due to Voigt [5] and Bátkai et al. [1]. In this talk we present a result on positive Miyadera–Voigt perturbations of bi-continuous semigroups, which generalize the result in [5]. As an application we discuss a explicit semigroup on the space $\mathrm{M}(\mathbb{R})$ of bounded Borel measures.

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The horofunction boundary of the infinite dimensional hyperbolic space

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In recent years the study of the geometry of the infinite dimensional real hyperbolic space $\mathbb{H}^\infty$ has gained significant momentum since it was popularised by Gromov. We will be using Klein's model of $\mathbb{H}^\infty$, which lies in the Lorenz cone, and use the order structure to give a complete description of the horofunction boundary of $\mathbb{H}^\infty$ and its Busemann points. Horofunctions are a fundamental tool in metric geometry and have found applications in numerous fields including, geometric group theory, ergodic theory, real and complex dynamics, nonlinear operator theory and metric and non-commutative geometry. The Busemann points are horofunctions that are the limits of so called almost geodesics introduced by Rieffel. These special horofunctions are known to be particularly useful in the studying isometric problems in metric spaces.

In search of asymmetric inner products

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An asymmetric norm on a real vector space X is a function $p : X \to [0, \infty)$ which is subadditive, positively homogeneous and such that $p(x) = 0 = p(-x)$ implies that $x = 0$. Although the theory of asymmetrically normed spaces is fairly well developed, there has been no attempt so far to define the notion of an asymmetric inner product, and it is not immediately clear what such a definition should look like. To get some idea about how to proceed, we look at an easier problem, that of finding a definition for an asymmetric semi-inner product. It is known that for every normed space, there is at least one semi-inner product defined on it which generates the norm of the space. There is a natural way to adapt the definition of a semi-inner product to yield an asymmetric version. We show that with this definition, it is still the case that on every asymmetrically normed space there exists at least one asymmetric semi-inner product which generates the asymmetric norm, and compare conditions ensuring uniqueness of the semi-inner product in the symmetric and asymmetric cases.

Lower Choquet envelopes and applications

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Let Ω be a finite set, let $f : \mathbb{R}^\Omega \to \mathbb{R} \cup \{+\infty\}$ and let $\underline{v}_f : 2^\Omega \to \mathbb{R} \cup \{-\infty, +\infty\}$ be the canonical set function defined by $\underline{v}_f(A) := \inf_{n\in\mathbb{N}} nf(\frac{1}{n}1_A)$ for all $A \subseteq \Omega$. We provide sufficient conditions on f so that it admits a lower Choquet envelope, that is, for all $x \in \mathbb{R}^\Omega$, $f(x) \geq \int_\Omega^C x\,d\underline{v}_f$, where $\int_\Omega^C$ denotes the Choquet integral.

If f is positively homogenous, non-decreasing, and normalized (i.e., $f(1_\Omega) = 1$), then $\underline{v}_f(A) := f(1_A)$ is the so-called non-additive probability (capacity) associated with f, called risk-neutral capacity in models of mathematical finance.

Our main result allows us to deduce the standard Choquet representation theorem characterizing functions f such that, for all $x \in \mathbb{R}^\Omega$, $f(x) = \int_\Omega^C x\,d\underline{v}_f$, i.e., the above inequality is replaced by an equality. Applications of the existence of the lower Choquet envelope of f are given in mathematical finance when f is a financial pricing rule.

Some results on Hausdoff operators

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Liflyand and Móricz studied the Hausdorff operator on the real Hardy space and proved the boundedness property and the commuting relations of this operator and Hilbert transformations. The main objective of this paper is extending these results to the two contexts of Dunkl theory and the Jacobi hypergroup.

This talk is based on the joint work with T.Kawazoe and F.Saadi.

Positive surjective isometries on non-commutative (quantum) symmetric spaces

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The disjointness preserving property of isometries between commutative (classical) L^p-function spaces plays a crucial role in facilitating a structural description of such isometries ([1]). It was later shown ([3]) that isometries between quantum L^p-spaces are also disjointness preserving and that the techniques employed in the classical setting can be adapted to obtain an analogous result in the quantum setting. It is therefore natural to consider if isometries between quantum symmetric spaces (generalizations of quantum L^p-spaces) are also disjointness-preserving and if so whether structural descriptions of such isometries can be obtained. It has recently been shown ([2]) that a positive isometry between a symmetric space and a fully symmetric space with K-strictly monotone norm is disjointness preserving and a structural description of such isometries has been obtained in the finite setting. In this talk we present some complementary results regarding positive surjective isometries between quantum symmetric spaces in the semi-finite setting.

This is joint work with Jurie Conradie from the University of Cape Town.

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Topological bicommutant theorems in vector lattices

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Suppose that F is a vector lattice. Then we let τ_0 be the topology on F generated by the subsets

$$O_{x_0,f,n^{-1},y} := \{x \in E : f(|x - x_0| \wedge y) < n^{-1}\}$$

of F for all $n \in \mathbb{N}$, $x_0 \in F$, $y \in F^+$, and positive order continuous functions $f : F \to \mathbb{R}$.

Let E and F be vector lattices, where F is Dedekind complete. For $x \in E^+$, define $\varphi_x : \mathcal{L}_b(E,F) \to F$ by setting $\varphi_x(T) := Tx$. Let τ' be the coarsest topology on $\mathcal{L}_b(E,F)$ such that φ_x is continuous for all $x \in E^+$, when F is supplied with the τ_0-topology.

Suppose that E is a Dedekind complete vector lattice. Then we let $\mathcal{L}_n(E)$ denote the Riesz algebra of all order continuous operators on E. We say that a subalgebra $\mathcal{A}$ of $\mathcal{L}_n(E)$ is a *band algebra* if $\mathcal{A}$ is also a band in $\mathcal{L}_n(E)$. We say that a band B in E is $\mathcal{A}$-*invariant* if $Tx \in B$ for all $x \in B$ and $T \in \mathcal{A}$, and that B is $\mathcal{A}$-*reducing* if both B and B^d are $\mathcal{A}$-invariant. A band algebra $\mathcal{A}$ in $\mathcal{L}_n(E)$ is said to have the $*$-*property* if every $\mathcal{A}$-invariant band in E is also $\mathcal{A}$-reducing. Let $\mathcal{A}' = \{S \in \mathcal{L}_n(E) : ST = TS \text{ for all } T \text{ in } \mathcal{A}\}$. Then we have the following topological bicommutant theorem in vector lattices, which is analogous to the topological version of von Neumann's bicommutant theorem in Hilbert spaces.

Thoerem. Let E be a Dedekind complete vector lattice such that, for all $x \in E^+$, there exists a strictly positive order continuous function f_x on $[0,x]$. Suppose that $\mathcal{A} \subseteq \mathcal{L}_n(E)$ is a unital band algebra with the $*$-property. Then $\mathcal{A}'' = \overline{\mathcal{A}}^{\tau'}$. If E is an order continuous Banach lattice, then $\mathcal{A}'' = \overline{\mathcal{A}}^{SOT}$. If E is atomic, then $\mathcal{A}'' = \mathcal{A}$.

This supplements earlier results by Björn de Rijk and Marcel de Jeu on the ordered analogue of the representation-theoretical version of the classical bicommutant theorem.

Positive commutators of positive operators

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It is known that a positive commutator $C = AB - BA$ between positive operators on a Banach lattice is quasinilpotent whenever at least one of A and B is a compact operator. We study the question under which conditions a positive operator can be written as a commutator between positive operators. As a special case of our main result we obtain that positive compact operators on order continuous Banach lattices which admit order Pelczyński decomposition are commutators between positive operators. Our main result is also applied in the setting of a separable infinite-dimensional

Banach lattice $L^p(\mu)$ $(1 < p < \infty)$.

This is joint work with Marko Kandić from University of Ljubljana.

A result about lattice homomorphisms

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Let A and B be f-algebras with unit elements e_A and e_B respectively. A positive operator T from A to B satisfying $T(e_A) = e_B$ is called a Markov operator. In this definition we replace unit elements with weak order units and, in this case, call T to be a weak Markov operator. Then we show that a weak Markov operator is a lattice homomorphism if and only if it is an extreme point in the collection of all weak Markov operators from A into B provided B is order complete.

This is joint work with Serkan Ilter from Istanbul University.

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Hyperstonean spaces associated with group actions

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In this project we look at when the Gelfand dual of the C^*-algebra consisting of the (right-) uniformly continuous functions on a Hausdorff group is hyperstonean. Or equivalently, when this algebra is a von-Neumann algebra. We also explore how this problem is related to the topological-dynamical aspects of Ramsey theory.

This is joint work with Jan Harm van der Walt from the University of Pretoria.

Difference property as a kind of stability

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We say that a function class $\mathcal{F} \subset \mathbb{R}^{\mathbb{R}}$ has the *difference property* provided that for any function $f : \mathbb{R} \longrightarrow \mathbb{R}$ satisfying the condition

$$\Delta_h f \in \mathcal{F} \quad \text{for all} \quad h \in \mathbb{R}\,,$$

there exists an additive function $a : \mathbb{R} \longrightarrow \mathbb{R}$ such that $f - a$ becomes a member of the class $\mathcal{F}$. Here Δ_h stands for the usual difference operator:

$$\Delta_h f(x) = f(x+h) - f(x), \quad x \in \mathbb{R},\, f \in \mathbb{R}^{\mathbb{R}}.$$

Numerous important function classes do enjoy this property. Surprisingly, the class of convex functions does not. Our main positive result states that the family of all delta-convex mappings (differences of two convex functions) has the difference property. As a proof tool the technique of $\mathbb{Q}$-differentiation will be applied.

Positive representations of algebras of continuous functions

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If X is a locally compact Hausdorff space, then a representation of the complex C*-algebra $\mathrm{C}_0(X)$ on a Hilbert space H is given by a spectral measure that takes its values in the orthogonal projections on H. It is natural to ask whether something similar is true for a positive representation of the ordered Banach algebra $\mathrm{C}_0(X)$ on a Banach lattice E. If E is a KB-space (e.g if E is an L^p-space for finite p, or if E is reflexive), then the answer is affirmative: the representation is given by a spectral measure that takes its values in the positive projections on X; see [1].

The proofs in [1] make use of the fact that E is a Banach space, but some results in [1] suggest that a purely order-theoretic more general approach might also be possible. In this lecture, we shall explain that this is indeed the case.

As a preparation, we shall sketch an integration theory for measures taking values in a suitable partially ordered vector space E. After that, we shall discuss a Riesz representation theorem for a positive map $T\colon \mathrm{C}_0(X) \to E$. Under mild conditions, this is given by a positive E-valued measure. In the next step, we apply the previous result to a positive representation $\pi\colon \mathrm{C}_0(X) \to A$, where A is a suitable partially ordered algebra. In that case, the pertinent positive A-valued measure takes values in the idempotents of A.

If A equals the regular operators on a suitable partially ordered vector space E, then the previous result yields a spectral measure for π that takes its values in the positive projections on E. This result has not only the main result in [1] as a special case, but also the aforementioned existence of spectral measures for representations of $\mathrm{C}_0(X)$ on a Hilbert space.

This is joint work with Marcel de Jeu.

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The countable sup property for vector lattices of continuous functions

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In this talk we will present sufficient and necessary conditions under which the vector lattice $C(X)$ and its sublattices $C_b(X)$, $C_c(X)$ and $C_0(X)$ have the countable sup property. It turns out that the countable sup property of $C(X)$ is tightly connected to the countable chain condition of the underlying topological space X. We will also address the question whether the vector lattice $C(X \times Y)$ has the countable sup property provided that $C(X)$ and $C(Y)$ both have it. Under the assumption of the continuum hypothesis, one can find a compact Hausdorff space X such that $C(X)$ has the countable sup property while $C(X \times X)$ fails to have it. We will provide some positive results in this direction.

This is joint work with Aleš Vavpetič from University of Ljubljana.

Relatively uniformly continuous semigroups on vector lattices

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We shall introduce and study the notion of relatively uniformly continuous operator semigroups on vector lattices endowed with the relative uniform topology. We will present some examples, basic results and a vector lattice property which is important for such semigroups.

This is joint work with Marko Kandić and Marjeta Kramar Fijavž from University of Ljubljana.

Orthogonality and absolute compatibility in operator algebras

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In this talk, we shall discuss the notions of *orthogonality* and *absolute compatibility* for a pair of positive elements in a C*-algebra. We shall present an order-theoretic characterization of algebraic orthogonality in C*-algebras. We shall also characterize absolute compatibility in the case of a von Neumann algebra and give a geometric description of this notion in the case of $\mathbb{M}_2$.

Vectorial Form of Ekeland Variational Principle with Applications to Vector Equilibrium Problems

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Our aim is to derive Ekeland type variational principles for bifunctions which take values in a vector space ordered by a convex cone, or more generally, by a so called *free disposal set*. We avoid to assume the demanding triangular inequality for bifunctions, a condition which often appears within result dealing with Ekeland principle for bifunctions. Then, by using such variational principle, we derive some existence results for solutions of vector equilibrium problems and vector quasi-equilibrium problems.

This is joint work with Suliman Al-Homidan (King Fahd University of Petrol and Minerals, Dhahran, Saudi Arabia) and Qamrul Hasan Ansari (Aligarh Muslim University, Aligarh, India).

The results are inspired by a previous work G. Kassay [1].

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Generation of relatively uniformly continuous semigroups

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We will present a Hille-Yosida type theorem on vector lattices. The theorem provides necessary and sufficient conditions for an operator to be the generator of an exponentially order bounded relatively uniformly continuous positive semigroup.

This is joint work with Michael Kaplin and Marko Kandić form University of Ljubljana.

Entropy for general quantum systems

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We review the concept of entropy, showing how that lead to the introduction of von Neumann entropy for B(H). We then show how Orlicz spaces associated to von Neumann algebras may be constructed for first the semifinite case, and then also the type III case, before going on to show how this Orlicz space structure may be used to in a very natural way extend the concept of von Neumann entropy to general von Neumann algebras. If time allows we will also discuss various ways of describing relative entropy in a von Neumann algebraic context, and how that relates to the entropy for single states mentioned above. In this discussion we will show how the theory of Haagerup L^p spaces may be used to complement and sharpen extant descriptions of relative entropy for quantum systems.

Universal separable Banach lattices

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We will show the following:

1. Every separable Banach lattice is lattice isometric to a sublattice of $C(\Delta, L^1)$.
2. There is a separable Banach lattice X so that every separable Banach lattice is lattice isometric to a quotient of X by a closed lattice ideal I.

Joint work with Lei Li (Nankai University), Timur Oikhberg and Mary Angelica Tursi (University of Illinois at Urbana-Champaign).

On p-compact operators and order boundedness

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Let $1 \leq p \leq \infty$. While there exist compact operators between Banach spaces which fail to be p-summing for all p, and there exist p-summing operators which fail to be compact, we'll show that in the factorization of members of the Banach operator ideal $\mathfrak{K}_p$ of p-compact operators defined in [1] the order boundedness of an adjoint factor taking values in a Banach lattice gives rise to p-integral operators. The imposition of equimeasurability on this factor even gives rise to p-nuclear operators in the sense of Pietsch [2]. This is interesting, given that $\mathfrak{K}_p = (\mathfrak{N}^p, \|\cdot\|_{\mathfrak{N}^p})^{sur}$, the surjective hull of the right p-nuclear operators since by definition, $(\mathfrak{N}^p, \|\cdot\|_{\mathfrak{N}^p}) = (\mathfrak{N}_{(p,1,p)}, \|\cdot\|_{\mathfrak{N}_{(p,1,p)}})$ [2], and so this is different from the classical Pietsch p-nuclear operators.

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Inverse Monotone Operators in Variational Formulations of Ellptic PDEs

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In this talk we discuss the properties of an elliptic differential operator which is commonly used in Partial Differential Equations (PDEs) arising in many areas, for example Mathematical Physics and Theoretical Biology. For classical solutions, that is, sufficiently smooth functions, the results regarding order (comparison) follow from the maximum principle. We derive similar results for the variational formulation. In the cases of both classical and variational formulations, we show that the comparison theorems can be recast in a general framework; namely, in terms of inverse monotone operators.

Properties of cones in Banach algebras

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Let A be a Banach algebra ordered by an algebra cone C. We will discuss how some fundamental properties of C carry over to algebra cones in Banach algbras contained in A and in quotient algebras involving A.

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Rational positive real odd matrix functions

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Positive real functions play a central role in electrical circuit theory, and more generally in the theory of passive systems, which includes certain aspects of control theory. These functions are right half-plane analytic, and form a convex cone in the space of rational functions. We consider the function class of rational positive real odd functions, **PRO** for short, that is, rational functions with real values on the real line whose real part is positive on the right half-plane and which is defined on the left half-plane by the property that it must be an odd function

$$f(-x) = -f(x).$$

The class of functions **PRO** forms a convex invertible cone, which is singly generated by the **PRO** function

$$f(z) = z.$$

Foster in [F24] identified **PRO** functions as the impedances of lumped electrical circuits generated by inductances and capacitors, and provided a description in terms of the poles of a **PRO** function f, which are all on the imaginary axis, simple, with positive residues and interlace with the zeros of f.

The question arises, which of the above results for scalar **PRO** functions extend to matrix **PRO** functions? By using realization theory for descriptor systems we provide partial analogues of the scalar-valued results.

This is joint work with Sanne ter Horst.

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Lattice-subspaces and positive bases, an application in big data.

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Suppose that A is a nonnegative $n \times m$ real matrix. In big data, the nonnegative matrix factorization problem (NMF problem) is the determination of two nonnegative real matrices F, V so that $A = FV$ with intermediate dimension p as small as possible and smaller than $min\{n, m\}$. The NMF problem has many applications in data analysis problems such as in chemical concentrations, document clustering, image processing, e.t.c. Since the exact NMF is not solvable in general, in applications the NMF problem is commonly approximated numerically. In this talk we present a general mathematical method for the determination of an exact NMF factorization of A, i.e. we determine two nonnegative real matrices F, V so that $A = FV$. This method is based on the theory of finite dimensional lattice-subspaces and positive bases expanded in [1] and [2]. During the first steps of this process the intermediate dimension p of F, V is determined and therefore we have an easy criterion for the applicability of our method. Also we give the matlab program for the computation of the nonnegative factors F, V of A.

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Dual of a special ordered cone

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Let $\mathcal{C}$ be a cone. As in [3], by using an order structure, the cone $\mathcal{C}$ is topologized. This ordered cone is called Locally Convex Cone. The collection of non-empty convex subsets of the cone $\mathcal{C}$ by an order is a locally convex cone again, which denoted by $Conv(\mathcal{C})$. In this paper, we study the dual of the mentioned locally convex cone.

This is a joint work with Amir Dastouri from the University of Tabriz.

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Removability results for subharmonic functions, for harmonic functions, for separately subharmonic functions, for separately harmonic functions and for holomorphic functions, a survey

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Blanchet [1] has shown that a $\mathcal{C}^2$ subharmonic function can be extended through a $\mathcal{C}^1$ hypersurface provided the function satisfies certain $\mathcal{C}^1$ type continuity conditions on the exceptional hypersurface. Measuring the exceptional sets with the aid of Hausdorff measure, we have recently improved Blanchet's result by giving related results for subharmonic functions, for harmonic functions and for holomorphic functions, see [5, 4] and the references therein. In addition to these already published results, we have very recently given new extension results for separately subharmonic functions and for separately harmonic functions, see [6, 7]. Now we give a survey of all of these recent results.

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Arens Extensions for Polynomials and the WoodburySchep Formula

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We characterize the order continuous component of a positive s-homogeneous polynomial with Dedekind complete range. In addition, we apply our results to Arens extensions of polynomials of order bounded variation.

This is joint work with Gerard Buskes from the University of Mississippi.

Order isomorphisms between order intervals in JB-algebras

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In this talk we will discuss the structure of order isomorphisms on cones and unit order intervals in unital JB-algebras.

Series and Power Series on Universally Complete Complex Vector Lattices

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We discuss an order-theoretic analogue to the theory of series and power series on complex Banach spaces. In particular, we introduce an nth root test for series as well as a Cauchy-Hadamard type formula for power series on universally complete complex vector lattices. These results illustrate how fundamental aspects of vector lattice theory, such as weak order units and projection bands, play a significant yet almost hidden role in the classical study of series and power series.

Discrete harmonic functions in Lipschitz domains

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We prove the existence and uniqueness of a discrete nonnegative harmonic function for a random walk satisfying finite range, centering and ellipticity conditions, killed when leaving a globally Lipschitz domain in $\mathbb{Z}^d$. Our method is based on a systematic use of comparison arguments and discrete potential-theoretical techniques. [1].

This is joint work with Sami Mustapha from Sorbonne University.

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Hardy inequalities in normal form

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Hardy inequalities come in many forms; continuous, weighted, discrete, with general measures, over starshaped sets in $\mathbb{R}^n$, and in more abstract settings. They give Lebesgue space estimates for operators like,

$$\int_0^x f(t)\,dt \quad \cdots \quad \sum_{k=1}^n f_k \quad \cdots \quad \int_{(-\infty,x]} f(t)\,d\lambda(t) \quad \cdots \quad \int_{\{t\in\mathbb{R}^n : t/|x|\in S\}} f(t)\,dt.$$

Inequalities involving the duals of these operators are also Hardy inequalities.

Every Hardy inequality can be transformed into *normal form*:

$$\left(\int_0^\infty \left(\int_0^{b(x)} f(t)\,dt\right)^q dx\right)^{1/q} \le C\left(\int_0^\infty f(t)^p\,dt\right)^{1/p}, \quad 0 \le f\downarrow.$$

The normal form parameter $b(x)$ is decreasing, $1 < p < \infty$ and $0 < q < \infty$.

The conversion to normal form is straightforward, it leaves the indices p and q unchanged, and it preserves the best constant (the operator norm.) Working with Hardy inequalities in normal form simplifies the proofs of many known results. New results proved for normal form inequalities readily translate back to results for the various other forms.

As usual, new results emerge from a new point of view. I will focus on new results for determining best constants, and for improving estimates of best constants.

An operator calculus for unbounded operators and holomorphic functional calculus for locally convex algebras

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In this talk, we provide a detailed exposition of the work of Taylor [1] in which he defines what is meant by the operator calculus on closed, unbounded linear operator in a Banach space X.

By $\mathcal{G}(T)$, we denote the family of all analytic functions f with domain $\Delta(f)$, for which $\sigma(T) \subseteq \Delta(f)$, $\Delta(f)$ contains a neighborhood of $\lambda = \infty$ and f is analytic at $\lambda = \infty$. Also, we define $f(\infty) := \lim_{\lambda\to\infty} f(\lambda)$.

A set $D \subseteq \mathbb{C}$ is called a *Cauchy domain* if it satisfies the following conditions: (i) D is open;
(ii) D has a finite number of components, the closures of any two of which are disjoint;
(iii) the boundary of D is composed of a finite number of closed rectifiable Jordan curves, no two of which intersect.

Using this, we define the following function, as given by Taylor:
When $f \in \mathcal{G}(T)$, define

$$f(T) = f(\infty)I + \frac{1}{2\pi i}\int_{+\partial(D)} f(\lambda)(\lambda\mathbf{1} - T)^{-1} d\lambda,$$

where I is the identity operator and D is a Cauchy domain such that $\sigma(T) \subseteq D$ and $\bar{D} \subseteq \Delta(f)$.

We briefly discuss the proofs as given by Taylor to show that this forms an operator calculus on unbounded, closed linear operators. We remark that the above-mentioned operator calculus is a generalization of the holomorphic functional calculus of $B(X)$, which is a Banach algebra, to unbounded operators, which do not form a Banach algebra.

By G.R. Allan [2], Taylor's argument can be modified somewhat to give a spectral theory and holomorphic functional calculus for locally convex algebras, by making use of the one-point compactification of $\mathbb{C}$. This modification of Allan, is also discussed.

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Complete positivity arising from couplings of quantum Markov semigroups

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A more general form of quantum detailed balance is given by the notion of balance, defined in terms of couplings between two quantum Markov semigroups exhibiting a kind of balancing behaviour. It can be shown that any coupling ω of two QMS naturally determines a completely positive map E_ω from the one QMS's von Neumann algebra to the other. In the talk it will be explained how completely positive maps like E_ω completely characterize the balance condition in a way that gives insight into the meaning of balance, and easily establishes important structural properties of the condition.

This is part of joint work with Rocco Duvenhage (UP, Pretoria).

Weighted Non-commutative Banach Function Spaces

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The theory of non-commutative Banach function spaces is by now a mature and highly refined field. Recently Labuschagne and Majewski, motivated by applications to Quantum Statistical Mechanics, introduced the concept of weighted non-commutative Banach function spaces.

We briefly illustrate the physical motivation for defining weighted non-commutative Banach function spaces and then give an overview of the description of these spaces. In describing these spaces we use a weighted analogue of a trace τ_x. By letting τ_x play the role of a trace, we can construct τ_x-measurable operators, a type of topology of convergence in measure, and a weighted analogue to the singular value function and can relate each concept back to their tracial versions.

Finally, we investigate the interpolation theory of weighted non-commutative Banach function spaces, and in particular describe the exact monotone interpolation spaces of the Banach couple $(L^1_x(\widetilde{\mathcal{M}}), L^\infty(\widetilde{\mathcal{M}}))$.

On closedness of law-invariant convex sets in rearrangement invariant spaces

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A well-known problem arising from the theory of risk measures asks whether order closedness of a convex set in a Banach lattice X guarantees closedness with respect to the topology $\sigma(X, X_n^\sim)$, where $X_n^\sim$ is the order continuous dual of X. This problem has a negative answer for some Orlicz spaces, but when the convex set is assumed to be law-invariant, it does have a positive answer for all Orlicz spaces. In this talk, we will show that the same result remains true for general rearrangement invariant spaces, that is, every order closed law-invariant convex set in a rearrangement invariant space X is $\sigma(X, X_n^\sim)$-closed. Some applications to quasiconvex law-invariant functionals will also be discussed.

Extending topologies to the universal σ-completion of a vector lattice

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We characterize the locally solid topologies that extend to the universal σ-completion of a vector lattice X. We then show that such topologies share many of the interesting properties of minimal topologies, but also exhibit subtle differences.

A Shmul'yan-type pre-order on matrix Herglotz functions

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Scalar Herglotz functions are analytic self-maps of the open right half-plane $\mathbb{C}_+$. They are sometimes also called Carathéodory or Nevanlinna functions. In the case of matrix Herglotz functions, the values on $\mathbb{C}_+$ are complex matrices whose real part is positive semidefinite. Such functions are important in various applications in electrical engineering and control theory. They also form a convex invertible cone.

Inspired by work of Yu.L. Shmul'yan [S80] on contraction operators, we introduce a pre-order structure on the cone of matrix Herglotz functions and consider its invariance under certain linear fractional transformations.

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On the linearity of order isomorphisms between cones with many engaged rays

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It is an intriguing phenomenon in the theory of partially ordered vector spaces that for some cones every order isomorphism is linear. It turns out that automatic linearity of order isomorphisms is related to the structure of the extremal rays of the cone. An extremal ray is said to be *engaged* if it lies in the linear span of the other extremal rays. A careful reworking of ideas of Noll and Schäffer (1978) yields that every order isomorphism $f\colon C \to K$, where C and K are Archimedean cones, must be linear on the positive linear span C_{E} of the engaged extreme rays of C. From there it follows that f is linear on an 'inf-sup hull' of C_{E}. The latter result is sufficiently strong to yield that every order isomorphism on the cone of positive semi-definite operators on a Hilbert space (of dimension at least 2) is linear, which is a result first proved by Molnár (2001) using different methods.

This is joint work with Bas Lemmens (University of Kent) and Hent van Imhoff (Leiden University).

For which ordered vector spaces are all order isomorphisms affine linear?

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We are interested in knowing, which pairs of ordered vector spaces have the property that all order isomorphisms between them are automatically affine? Here, an order isomorphism means a bijection that is order preserving in both directions.

In finite dimension, it is known that this property holds as long as the cone of positive elements of each space does not have a 1-dimensional factor. There are extensions of this result to infinite dimension, but they typically make strong assumptions on the set of extreme vectors of the cone. However, many interesting spaces have cones with few or no extreme vectors. Our approach is to instead look at the dual cone. Our setting is that of order unit spaces that are complete with respect to their order unit norm. We present a necessary and sufficient condition, in terms of the geometry of the dual cone, for all order isomorphisms between two such spaces to be affine.

Applications of generalized B*-algebras to quantum mechanics

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It is well known that observables in quantum mechanics, such as position, momentum, and the Hamiltonian, are self-adjoint unbounded operators on a Hilbert space. More generally, one can consider such observables to be self-adjoint elements of a $*$-algebra. A special class of locally convex $*$-algebras, which can be faithfully represented as $*$-algebras consisting of unbounded linear operators on a Hilbert space, are generalized B*-algebras (GB*-algebras for short). They were first studied by G.R. Allan and P.G. Dixon in (1) and (2) during the late sixties to early seventies, and are generalizations of C*-algebras.

In this talk, we make the case for linearly nuclear GB*-algebras as appropriate locally convex $*$-algebras of which its self-adjoint elements are regarded as observables appearing in quantum mechanics (by linearly nuclear, we mean nuclear as a locally convex space. This is not to be confused with the notion of nuclear GB*-algebra, introduced in (3)). We start by putting linearly nuclear GB*-algebras in the context of the rigged Hilbert space framework, followed by how one can study quantum entanglement of states within the setting of linearly nuclear GB*-algebras. Our investigations of quantum entanglement led us to results on pure states and integral representations of states of GB*-algebras, which we will also discuss. We end this talk with an extremal decomposition theorem for states on linearly nuclear locally convex quasi $*$-algebras, and discuss the relevance of this to quantum mechanics.

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On unbounded Dunford-Pettis operators between Banach lattices

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Suppose E is a Banach lattice. A net (x_α) in E is said to be **unbounded absolutely weakly** convergent to $x \in E$ (uaw-convergent, for short) if for each positive $u \in E$, one has $|x_\alpha - x| \wedge u \xrightarrow{w} 0$; in notation $x_\alpha \xrightarrow{uaw} x$. In a recent paper, the author considered this definition and investigated several properties of it. In particular, order continuous Banach lattices and reflexive ones have been characterized in term of this convergence. Observe that this definition is, in a sense, an unbounded version of weak convergence. Motivated by this, a linear operator $T : E \to X$, where E is a Banach lattice and X is a Banach space, is called **unbounded absolute weak Dunford-Pettis** (uaw-Dunford-Pettis, in brief) if for every bounded uaw-null sequence $(x_n) \subseteq E$, $(T(x_n))$ is norm null; we denote by $B_{UDP}(E)$ the space of all uaw-Dunford-Pettis operators on a Banach lattice E. In this talk, we investigate some important aspects of this class of operators;

1. $B_{UDP}(E)$ forms a closed subalgebra of the algebra of all continuous operators.
2. Every uaw-Dunford-Pettis operator is M-weakly compact.
3. If E is order continuous, then every positive M-weakly compact operator is also uaw-Dunford-Pettis.
4. If E is order continuous, then the square of every positive uaw-Dunford-Pettis (M-weakly compact) operator is compact.
5. In addition, we investigate the relation between this class of operators and L-weakly compact ones.

This is joint work with Nazife Erkursun-Ozcan from Hacettepe University and Niyazi Anil Gezer from Middle East Technical University.

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Actions of Amenable Groups and the Ergodic Decomposition

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Let G be a locally compact separable metric group, let m_L be a left Haar measure on G, assume that G is amenable and let $\alpha = (F_n)_{n\in\mathbb{N}}$ be a tempered Følner sequence of compact subsets of G of nonzero Haar measure. (All specialized topological group theory terms (like amenable groups and tempered Følner sequences) and terms in ergodic theory will have their definitions recalled during the talk.)

Now, let (X, d) be a locally compact separable metric space, and let $w : G\times X \to X$ be a left action which is jointly measurable and continuous in the first variable.

Let $C_0(X)$ be the Banach space of all real-valued continuous functions on X that vanish at infinity along the compact subsets of X.

Given an ordered triple (G, w, X) where G, w, and X are as above, one can define an ergodic decomposition of X in terms of the convergence properties of the sequences $\left(\frac{1}{m_L(F_n)}\int_{F_n} f(gx)\,\mathrm{d}m_L(g)\right)_{n\in\mathbb{N}}$, $f \in C_0(X)$, $x \in X$.

An ergodic decomposition of X is a partition of the space X which has the property that every w-invariant ergodic probability measure on X is concentrated on an element of the partition.

Our goal in the talk is to describe the ergodic decomposition defined by the convergence properties of the above sequences.